Math 70: 3.8 - 2nd Piecewise Functions

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Objectives: 1) Graph piecewise - defined functions.

To do this we need several smaller objectives:

a) Understand notation used for piecewise functions.

b) Evaluate piecewise functions

c) Determine y-values at endpoints of piecewise functions.

d) Determine if endpoints should be graphed with ● or O.

GC 21: Graphing Piecewise Functions

Exam 1 Information

An example of a piecewise-defined function, also called a piecewise function:

$$f(x) = \begin{cases} 2x+3 & \text{if } x \le 0 \\ -x-1 & \text{if } x > 0 \end{cases}$$

Note: The word "if" is optional

f(x) is the name of this function.

The brace { tells us it's piecewise.

(2x+3) and (−x−1) are the two pieces.

$x \le 0$ is the domain for the first piece (2x+3).

$x > 0$ is the domain for the second piece (−x−1)

Note: It has one name because it's all one function. The resulting graph MUST PASS THE VERTICAL LINE TEST!

1. Using f(x) above, evaluate.
   a) f(3)
   b) f(1)
   c) f(0)
   d) f(−1)

① continued.

a) $f(3)$

step 1: Identify which domain is true for this x.

$x=3$ is greater than 0 $\Rightarrow$ $x>0$.

step 2: Use the piece which applies to that domain. (Ignore the other piece!)

$-x-1$ if $x>0$.

step 3: Evaluate the appropriate piece.

$$\begin{aligned} & -x-1 \\ =\ & -(3)-1 \\ =\ & -4 \end{aligned}$$

$\boxed{f(3) = -4}$

b) $f(1)$

step 1: (as above) $x=1$ is $x>0$.

step 2: $-x-1$ if $x>0$

step 3: $-(1)-1$

$\boxed{f(1) = -2}$

c) $f(0)$

step 1: $x=0$ is $x\le 0$

step 2: $2x+3$ if $x\le 0$

step 3: $2(0)+3$

$\boxed{f(0)=3}$

d) $f(-1)$

step 1: $x=-1$ is $x\le 0$

step 2: $2x+3$ if $x\le 0$

step 3: $2(-1)+3$

$\boxed{f(-1)=3}$

② Using the same f(x), evaluate to complete the tables.

Hint: Use GC table function.

x	$f(x)$
-5	
-4	
-3	
-2	
-1	
0	

x	$f(x)$
3	
2	
1	
$1/2$	
$1/4$	
$1/8$	

Solution:

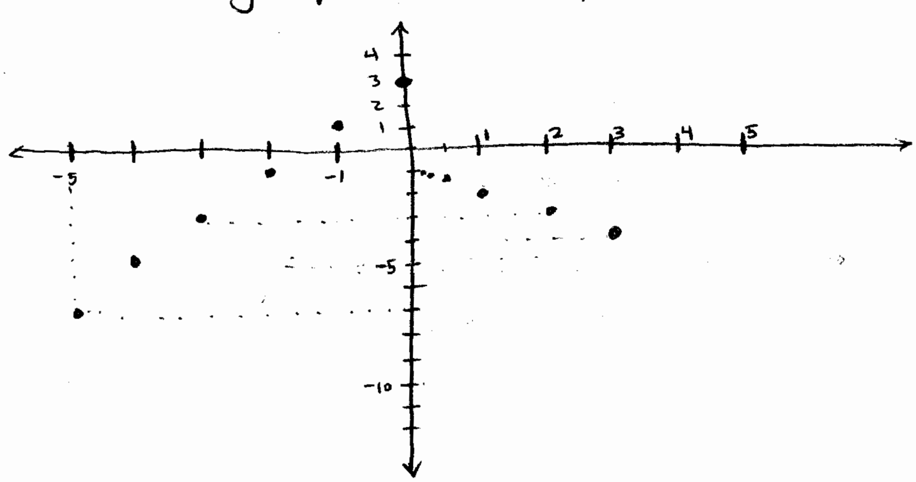
x	$f(x)$
-5	-7
-4	-5
-3	-3
-2	-1
-1	1
0	3

use $2x+3$ piece

x	$f(x)$
3	-4
2	-3
1	-2
$1/2$	-1.5
$1/4$	-1.25
$1/8$	-1.125

use $-x-1$ piece

③ Sketch the graph of the points we know so far.



(4) Complete the graph by

- calculating endpoints of each piece
- using ● to graph piece where endpoint is included ($x \le 0$ includes $x=0$).
- using o to graph piece where endpoint is not included ($x > 0$ does not include $x=0$).
- connecting dots within each piece

The endpoints occur when $x=0$, from domains.

$f(0) = 3$ is the ● endpoint of the piece $2x+3$.

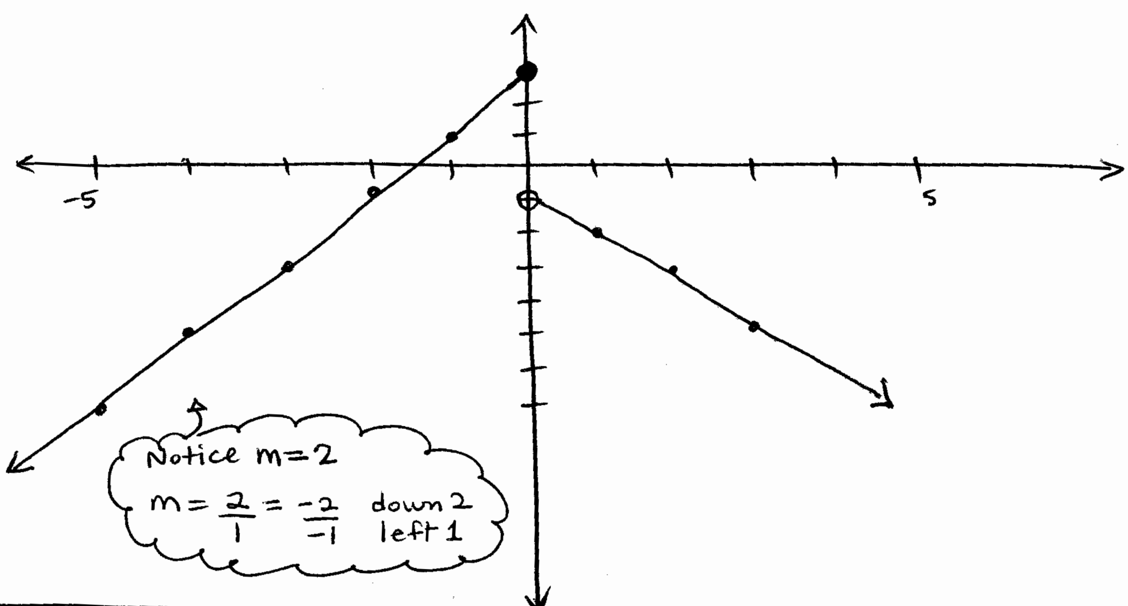
the endpoint of the piece $-x-1$ is also when $x=0$.
But this endpoint is not included.
Substitute $x=0$ to get y-coordinate.

$$-0-1 = -1$$

Graph $(0,-1)$ using open circle o.

Connect all the $x \le 0$ pts together.
Connect all the $x > 0$ pts together.

Note: The left piece may or may not connect to the right piece.



Note: Do NOT put arrowheads at the endpoints where domains are divided.

Note: Do put arrowheads on the ends where $x \to +\infty$ or $x \to -\infty$.

(5) Sketch graph of $f(x) = \begin{cases} -2x+4 & x \le -1 \\ 3 & x > -1 \end{cases}$

<u>step 1</u>: $-2x+4$ is a line with slope -2

3 is a horizontal line

<u>step 2</u>: Endpoints occur when $x = -1$

$f(-1) = -2(-1)+4 = 6$

plot $(-1, 6)$ using • included endpt.

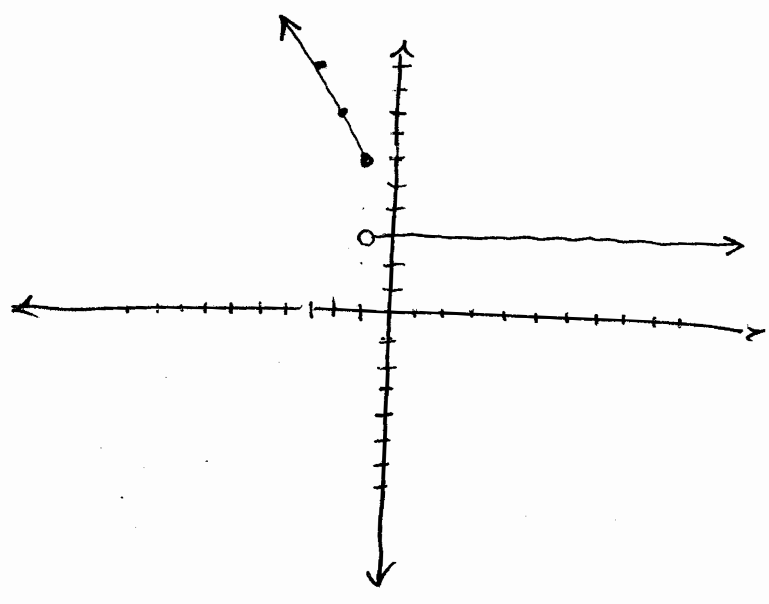
$3 \Rightarrow$ plot $(-1, 3)$ using o excluded endpt.

<u>step 3</u>: To graph $-2x+4$, $x \le -1$

plot $(-1, 6)$, use $m = \frac{-2}{1} = \frac{2}{-1} =$ 2 up 1 left.
using •

To graph 3, $x > -1$

plot $(-1, 3)$ using o, draw horizontal.



Yes, it passes the V.L.T!

<u>Helpful Hint</u>: For the $x < \#$ or $x \le \#$ piece, write slope as fraction with negative denominator. (go left).

For the $x > \#$ or $x \ge \#$ piece, write slope as fraction with positive denominator (go right)

Procedure for graphing piecewise functions with linear pieces.

$$f(x) = \begin{cases} ax + b & x \le c \\ dx + e & x > c \end{cases}$$

1) draw axes.
2) identify $x=c$ on graph, left & right pieces
$x \le c$ $x > c$
(or $x < c$ & $x \ge c$)

3) Subst $x=c$ into 1st piece
use ● if $x \le c$ or $x \ge c$
use o if $x < c$ or $x > c$

4) use slope of first piece to fill in line in correct direction
- check domain
- check sign of slope

5) repeat steps 3 & 4 for 2nd piece.

6) check that final graph passes VLT,

Now that we have explored a problem in detail, let's write our method.

<u>Step 1</u>: Identify the shape of each piece. [In M70, usually lines.]

<u>Step 2</u>: Evaluate endpoints in both pieces. Determine which is ● and which is ○ on graph.

<u>Step 3</u>: Graph each piece.

<u>Step 4</u>: Check that final graph passes V.L.T.

(5) Sketch graph of $f(x) = \begin{cases} x+2 & \text{if } x < 1 \\ 2x+1 & \text{if } x \geq 1 \end{cases}$

<u>Step 1</u>: $x+2$ is a line $y = x+2$ with slope 1.
$2x+1$ is a line $y = 2x+1$ with slope 2.

<u>Step 2</u>: Endpoints occur when $x = 1$.

$f(1) = 2(1) + 1 = 3$

plot $(1,3)$ using ● included endpt

In other piece $1+2 = 3$

plot $(1,3)$ using ○ excluded endpt

Note: In this graph, the endpoints overlap!

<u>Step 3</u>: To graph $x+2$, $x<1$

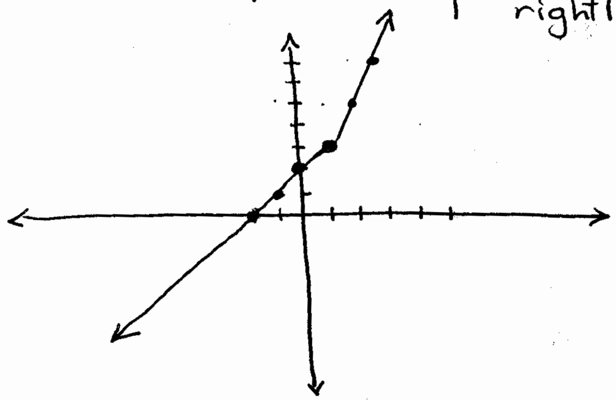
plot $(1,3)$ .... with ○ excluded endpoint

use slope $m = \frac{1}{1} = \frac{-1}{-1}$ down 1, left 1

To graph $2x+1$, $x \geq 1$

Plot $(1,3)$ with ● included endpoint

use slope $m = \frac{2}{1}$ up 2, right 1



Yes! It passes the VLT!

Extra Practice: Sketch graphs

(7) $f(x) = \begin{cases} x-1 & x \le 3 \\ -x+5 & x > 3 \end{cases}$

(8) $f(x) = \begin{cases} 4x-4 & x < 2 \\ -x+1 & x \geq 2 \end{cases}$

(9) $g(x) = \begin{cases} -3x & x \le -2 \\ 3x+2 & x > -2 \end{cases}$

(10) $k(x) = \begin{cases} -1 & x < -3 \\ 2 & x \geq -3 \end{cases}$

(11) Challenge question

$m(x) = \begin{cases} -x+2 & x < -1 \\ -x^2+4 & x \geq -1 \end{cases}$

M70

Solutions to Extra Practice

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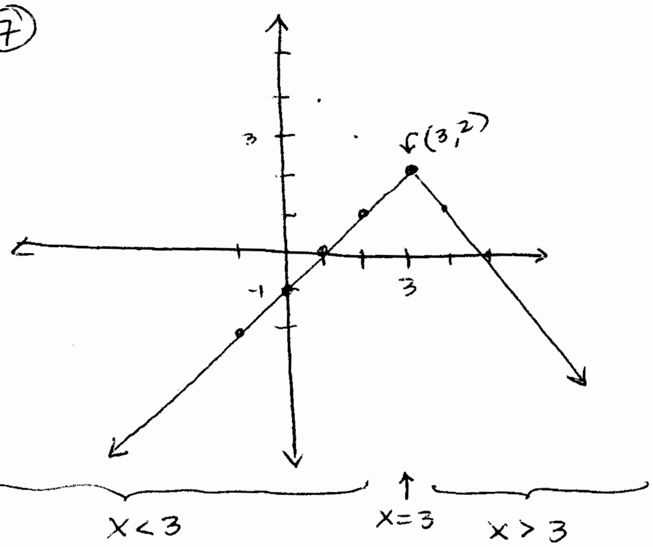


Note:

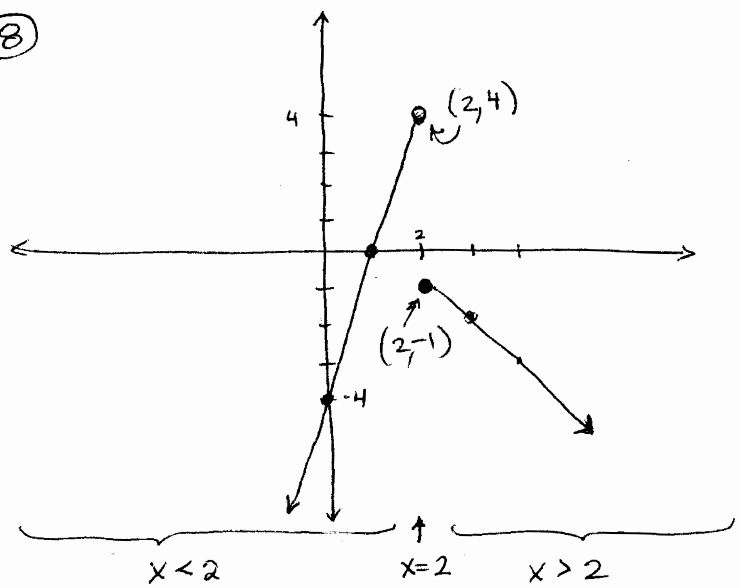
$$f(x) = \begin{cases} x-1 & x \le 3 \\ -x+5 & x > 3 \end{cases}$$

is the same as

$$f(x) = -|x-3| + 2.$$

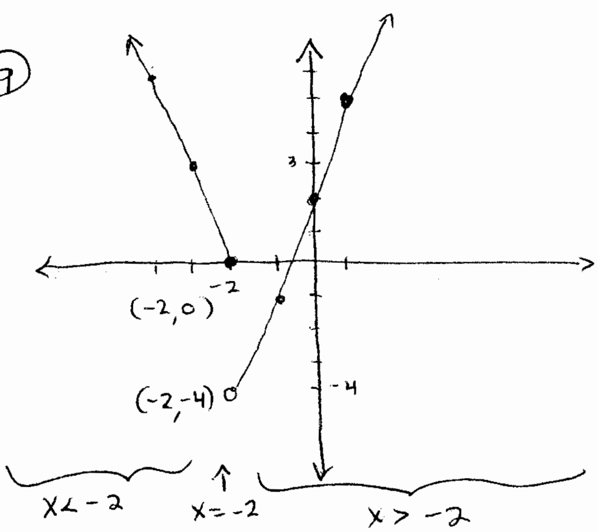
ALL absolute functions can be written as piecewise functions.

8



$$f(x) = \begin{cases} 4x-4 & x < 2 \\ -x+1 & x \ge 2 \end{cases}$$

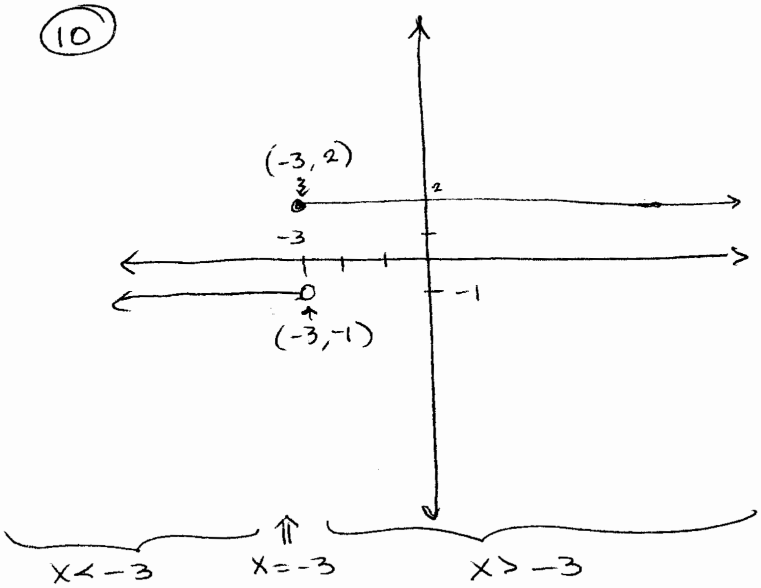
9



$$g(x) = \begin{cases} -3x & x \le -2 \\ 3x+2 & x > -2 \end{cases}$$

M70 5/6 2.7

(10)

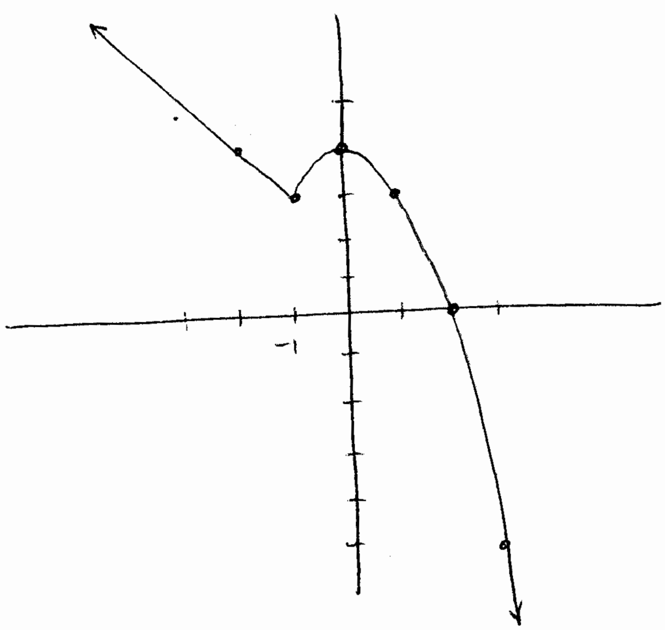


$$k(x) = \begin{cases} -1 & x < -3 \\ 2 & x \geq -3 \end{cases}$$

(11) $m(x) = \begin{cases} -x+2 & x < -1 \\ -x^2+4 & x \geq -1 \end{cases}$

lines

parabola opening down
vertex at (0,4)



$m(-1) = -(-1)^2+4 = 3$

(-1,3) plotted •

other piece $-(-1)+2 = 3$.
they connect at (-1,3).

Rev 1-15-16

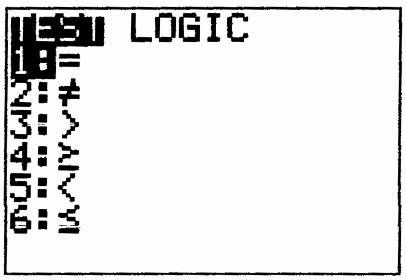
Name ______________________________

Date ______________________________

TI-84+ GC 21 Graphing Piecewise Functions

Objectives: Use TEST to compare a single value of a variable to a test value
Use TEST to compare many values of a variable to a test value
Multiply a function by a TEST result to limit domain and graph a piece
Determine if your GC will show $y = 0$ when multiplied, use divide by a TEST result
Add pieces to graph a piecewise function

The TEST menu is the second function above MATH: 



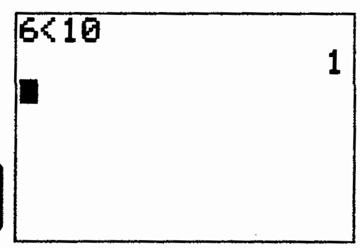
We can select from the list by typing the corresponding number. "Less than" is option 5, for example.

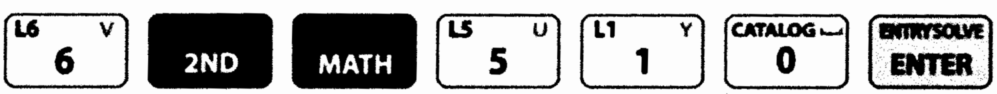
Example 1: Use GC to determine if

a) $6 < 10$

b) $6 \le 3$

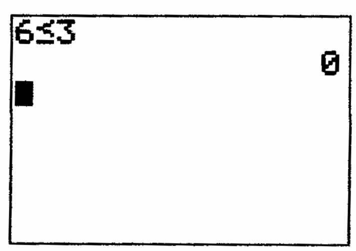
a) Use the TEST menu to determine if $6 < 10$





The GC says 1 if the answer to the question is true (or yes).
$6 < 10$ is a true statement, so GC says 1.

b) Use the TEST menu to determine if $6 \ge 3$

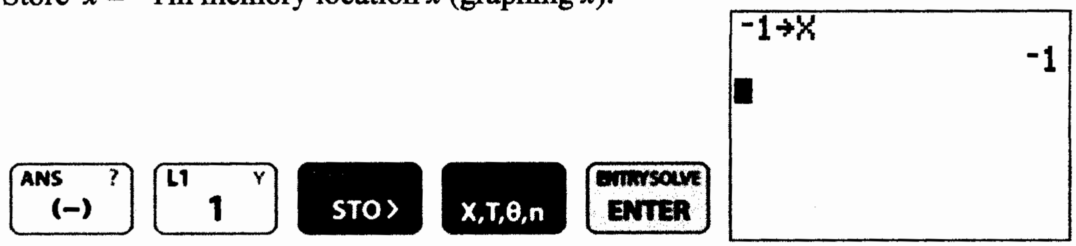


The GC says 0 if the answer to the question is false (or no).
$6 \ge 3$ is a false statement, so GC says 0.

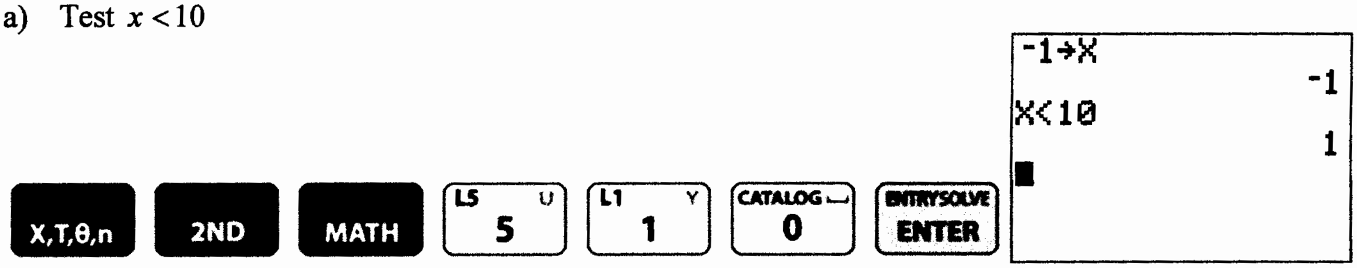
Example 2: Use GC to store $x = -1$ in memory and determine if

a) $x < 10$
b) $x \geq 3$

Store $x = -1$ in memory location x (graphing x).

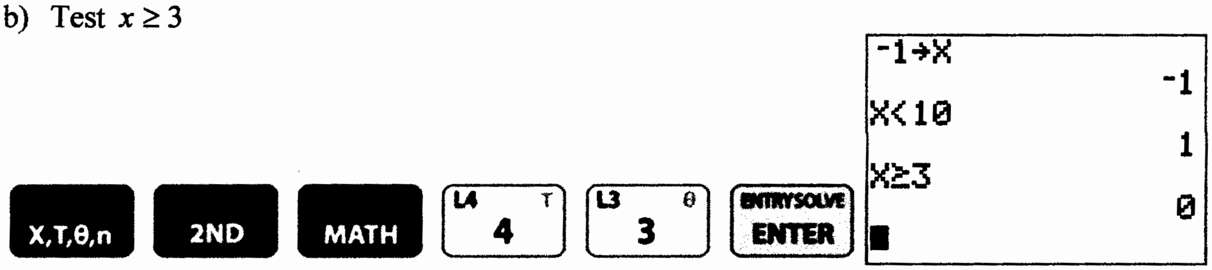


a) Test $x < 10$



The GC tests the value stored in memory, and concludes that yes, $-1 < 10$.

b) Test $x \geq 3$

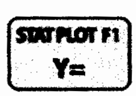


The GC tests the value stored in memory, and concludes that no, $-1 \geq 3$ is a false statement.

Example 3: Graph the TEST results for

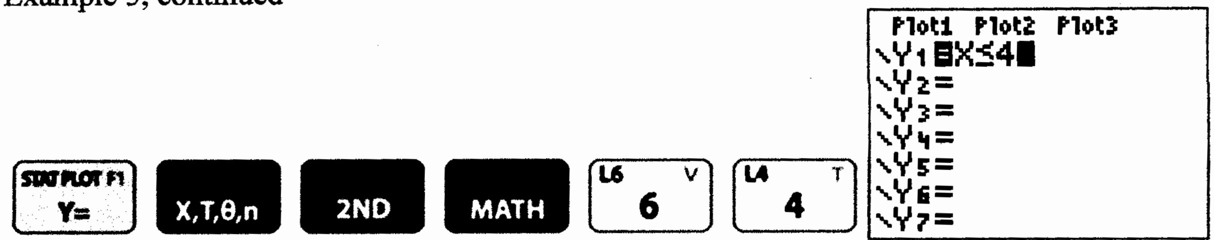
a) $x \leq 4$
b) $x > -2$

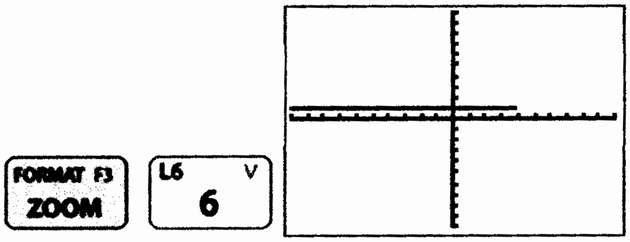
CAUTION: We are NOT graphing the inequality $x \leq 4$, which would be graphed on a number line. TEST gives y-coordinate that is 0 or 1, depending on whether x is less than or equal to 4 (y-coordinate 1) or not less than or equal to 4 (y-coordinate 0). The GC automatically generates the values of x from Xmin and Xmax in WINDOW.

a) Type the test $x \leq 4$ as a function in the 

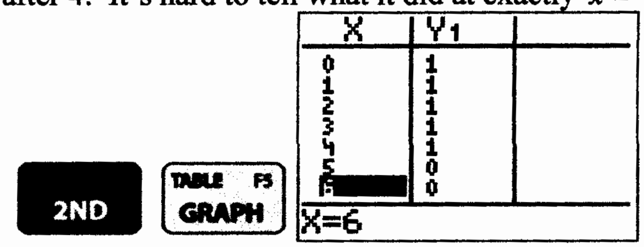
menu, then graph in a standard viewing window.

Example 3, continued

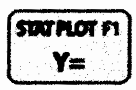




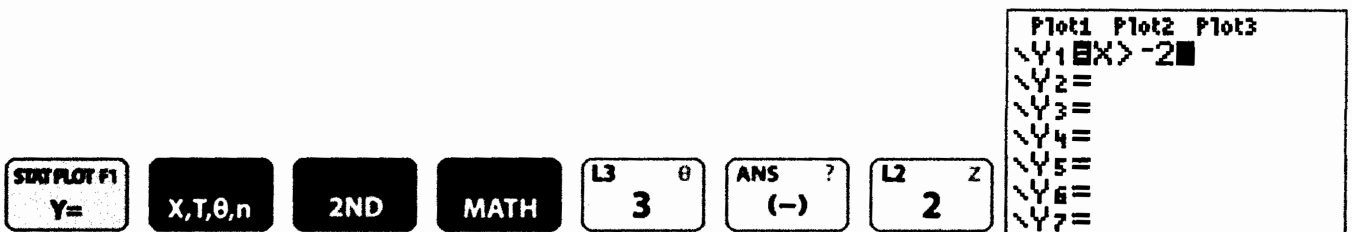
The GC graphs a horizontal line $y=1$ for all x values up to $x=4$, then a horizontal line $y=0$ for all x-values after 4. It's hard to tell what it did at exactly $x=4$ without checking a table.

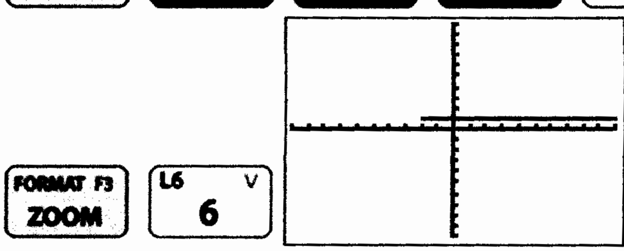


$x=4$ has y-value 1.

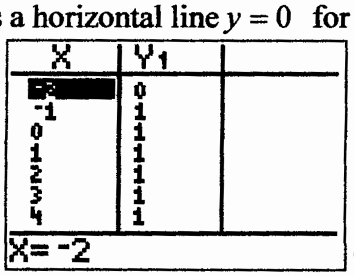
b) Type the test $x>-2$ as a function in the 

menu, then graph in a standard viewing window.





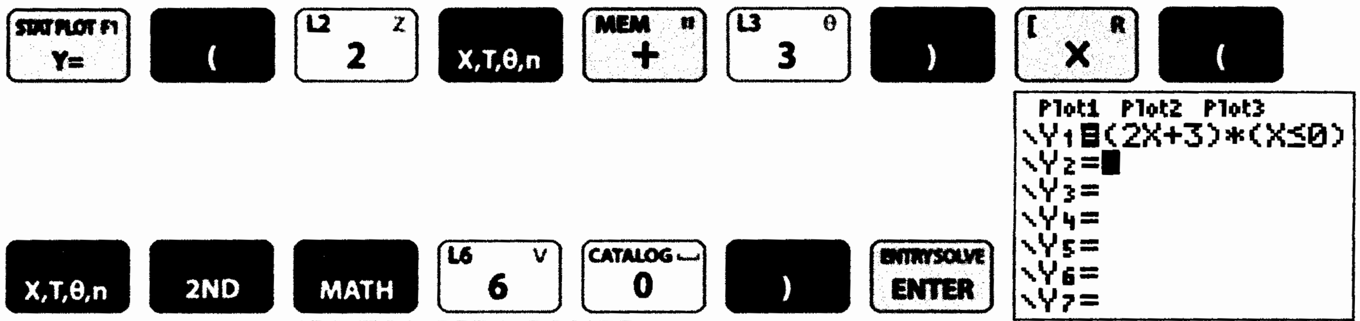
The GC graphs a horizontal line $y=0$ for all x values before $x=-2$, then a horizontal line $y=1$ for all x-

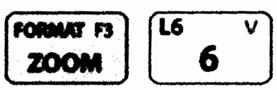


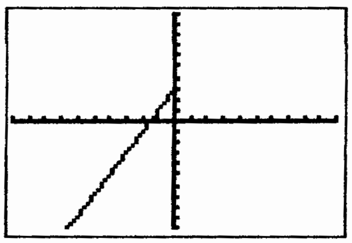
values after -2. The table shows that $x=-2$ is assigned y-coordinate 0.

Example 4: Graph the function $f(x) = 2x + 3$ for the domain $x \le 0$ by multiplying by the TEST results for its domain.

CAUTION: We want the function value times the test value, so we need to include additional parentheses!

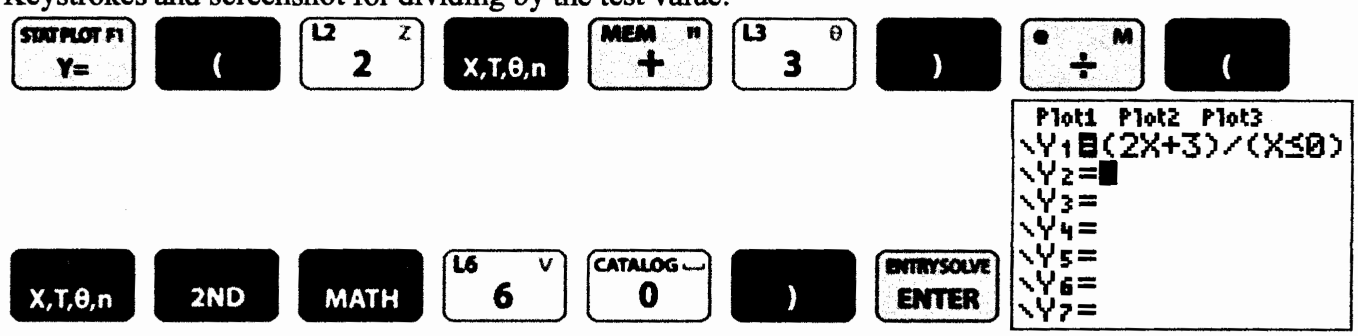






CAUTION: If you have a new color calculator, you may see a horizontal line $y = 0$ for the x-values on the right side of the graph. You can remove this by *DIVIDING* by the test value. Newer calculators do not graph values that result in $\div 0$.

Keystrokes and screenshot for dividing by the test value:

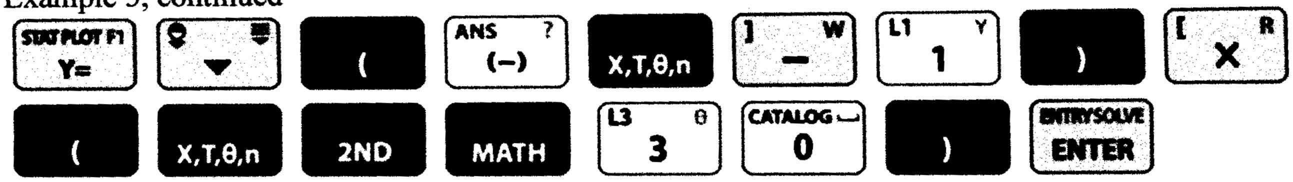


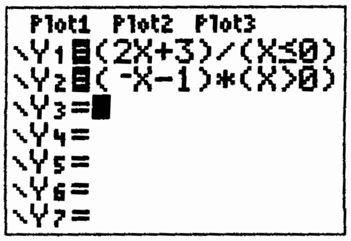
CAUTION: However, if you have an older calculator, dividing by the test value will completely confuse your calculator and it may graph nothing at all!

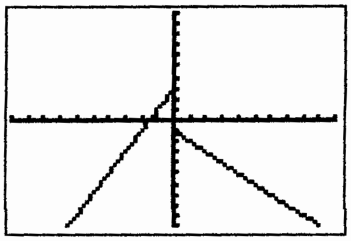
Example 5: Graph the piecewise function $f(x) = \begin{cases} 2x+3 & x \le 0 \\ -x-1 & x > 0 \end{cases}$ **by multiplying each piece by TEST results for its domain using two lines of the y= menu.**

The first piece is the same as Example 4. We continue with the second piece in y_2:

Example 5, continued





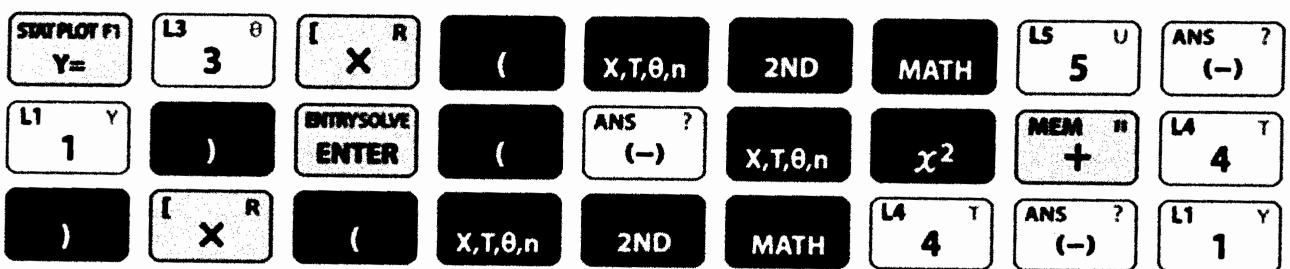


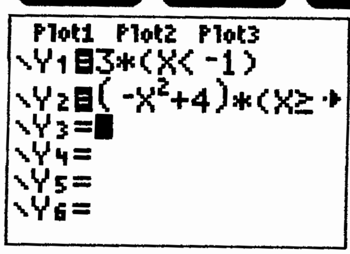
CAUTION: When you transfer this graph to paper, you must clearly indicate which piece includes its endpoint (closed circle) and which piece does not (open circle)!

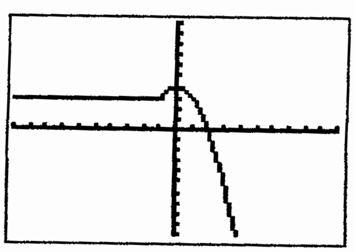
REMEMBER: A piecewise function is still a single function, and the resulting graph must pass the vertical line test!

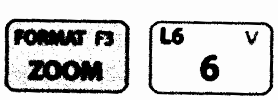
Example 6: Graph the piecewise function $f(x)=\begin{cases}3 & x<-1\\ -x^2+4 & x\geq -1\end{cases}$.

We expect the first piece to be a horizontal line while the second piece is a parabola.









Math 70 Exam Instructions

Ms. Carey

Instructions for all questions:

1. Write exact (not approximate) answers unless instructed otherwise.
2. Fully simplify all answers.
3. If units are given in the question, write appropriate units with your answer.
4. Answers with little or no work will receive little or no credit.
5. When sketching graphs on provided graph paper, draw axes in a suitable location, label the scale (if it's not 1), plot points and sketch curves neatly, and extend the graph accurately to the edge of the grid.
6. If you solve a problem using only your calculator, you must give a brief explanation, including essential keystrokes, of what you did on your calculator or you will not receive any credit.
7. If graphs have holes, jumps, or gaps, plot clearly whether endpoints are included (closed circles) or excluded (open circles).
8. Put a box around all final answers except graphs.
9. Scratch paper will not be graded.
10. Write correct notation at all steps of work.

And what they mean:

1) Do not round or approximate any answer unless the instructions explicitly say so. Exact answers include fully-reduced fractions, simplified square roots.
   a) If you put an irrational square root into your GC, then used >Frac, you've been tricked by your GC, which converted a decimal approximation of an irrational number into a rational number.
   b) Sometimes students get approximate answers by using a GC to analyze the graph, rather than demonstrating they have mastered the algebra objective being tested by the question.
2) Reduce all fractions, simplify all radicals, and know when to leave an answer factored or when to distribute and combine like terms.
3) Examples of units include "feet", "seconds", "feet per second", "liters", "liters per minute", or "squirrels".
4) Some problems can be accurately approximated using a graphing calculator, but skip the algebra objective being tested. Answers without work are assumed to be either
   a) Cheating
   b) Inappropriate use of a GC
   c) Random stabs in the dark by a student who didn't study sufficiently.
   d) All of the above.
5) The details of graphing are important.
6) An answer without work might have been copied from your neighbor! Product your reputation by showing work demonstrating your knowledge. Sometimes that knowledge is how to use your graphing calculator.
7) The details of graphing are important.
8) If I can't find your answer, or you have multiple attempts or answers without boxes, you're likely to earn very little credit. Be professional.
9) Show your work on the exam.
10) Use brackets [ ] or parenthesis ( ) or braces { } for their specific meanings. These are not interchangeable! In particular, the notation your graphing calculator uses may sometimes be computer syntax, but not correct mathematical notation.

Math 70 EXAM #1 Topics List

Math 70-33: Exam #1 Wednesday, 21 February, 1:20 PM – 2:25 PM
Math 70-34: Exam #1 Tuesday, 20 February, 1:20 PM – 2:25 PM

Optional Exam 1 Review session / office hours for any Math 70 student in either section:
Thursday, February 15, 3:30 PM - 4:30 PM, in the ASC, room 420.

Note: SWC has holidays Friday, February 16 and Monday, February 19. Campus tutoring will be closed.

Martin-Gay		**Your book: Bittinger**
Lecture sections	**Lesson number**	**Lecture sections**
Day 1 - Get a GC!	1	Day 1 - Get a GC!
1.6	2	2.4 & 2.5
1.6 & 1.3	3	5.2 & 1.8 - GC
1.7 & 1.8	4	2.3 - GC
2.1, App D, 2.3	5	3.1, 3.2 & 3.3*
2.4	6	3.4 & 3.5
2.5	7	3.6 & 3.7
2.2	8	3.8-1st
2.7	9	3.8-2nd*
		EXAM 1 (c1-2-3)

If you are repeating Math 70 or study with students in other sections of Math 70, it may be helpful to know how the other textbook, Martin-Gay, aligns with your textbook, Bittinger.

Lesson 9

Math 70: Review Problems Chapters 1-2-3

This is NOT a comprehensive review of every problem. It's a list of problems many past students should have studied more.

Some applications:

1) Margaret earns \$1500 per month, plus 5% commission.
   a. Complete the table:

Sales	\$8000	\$9000	\$10000	\$11000	\$12000
Monthly pay					

   b. If she sells \$11000 per month, what is her annual pay?
   c. If she needs to earn \$2200 per month, how much must she sell?
2) By 2018, the number of network analysts is expected to be 357,000. This is a 55% increase over 2008. How many analysts were there in 2008?
3) Find the amount of money in an account after 10 years if \$2500 is invested at 3.5% interest compounded quarterly?
4) A manufacturing company observes that the number of days lost due to injury or sickness increased linearly from 21 in 2008 to 59 in 2011. How many days were lost due to injury or sickness in 2010?
5) China, U.S., and France are predicted to be the top three tourist destinations by 2020. The U.S. is predicted to have 9 million more tourists than France, and China is predicted to have 44 million more than France. If the total number of tourists is predicted to be 332 million, find the number predicted for each country in 2020.

Some algebra (sometimes with GC):

6) Evaluate $0.2x^3 + 5x^2 - 6.2x + 3$ if $x = -3.1$
7) Simplify
   a. $0.2a - \frac{3}{8}b + \frac{1}{6}a - 0.75b$
   b. $(8.3x - 2.9y) - \left(9.6x - \frac{15}{7}y\right)$
8) Solve for x
   a. $Q = xt^2 + vt$
   b. $Q = xt^2 + xvt$

Some geometry:

9) Find the area and circumference of a circle with diameter 7.2 feet. Give exact and approximate answers (round to the nearest hundred-thousandth), and label which is which.
10) Find the volume of a cone if the radius is 6 inches and the height is 2.75 inches. Give exact and approximate answers (round to the nearest hundredth), and label which is which.
11) Find the area and perimeter of a parallelogram which side lengths $8x$ and $x+3$ and height $x+2$ (which is perpendicular to side length $8x$), where all measurements are in centimeters.

12) Find the area and perimeter of a trapezoid with base lengths $21x$ and $15x$, height $6x$, and remaining side lengths $8x$ and $7x$, where all measurements are in meters.

13) A circular dog pen has a circumference of 78.5 feet. Estimate how many dogs can be safely kept in the pen if each dog needs at least 60 square feet.

Things Math 70 students should know about exams:

1) You must come to an exam ready to show <u>what you know</u>, not ready to figure out something you saw before but never did correctly yourself, or never did correctly without help.
2) Alternate looking at hard problems (even if you don't finish) and doing the problems you <u>know</u> first to get your guaranteed points. Your brain's diffuse mode of learning can work on the problems while you're focused elsewhere. When you return to the problem, you'll likely remember more.
3) Anything that appeared on a PQ or a class handout (including GC handouts) is unquestionably a fair exam question.
4) Every topic and objective will be covered.
5) The GC is a tool you are supposed to be comfortable using because you've practiced all the keystrokes. The exam is not the time to wonder how to do something on your GC.
6) *All the GC skills from chapters 1-2-3 may (and probably will) be included on PQs or exams at any time during the rest of the term.
7) If you don't usually eat breakfast, please find a way to eat a healthy meal before an exam.
8) If you don't have a photo ID on the day of the exam, plan to bring yours the next exam, or at some other point if exam day is too stressful. Your SWC ID is sufficient, as is any document from any official issuer showing that your face and your name are the same as the face I see in class and the name I have on the roster.
9) When you are doing homework is the time to refine your learning
   a. First, learn to do the problem, no matter how long it takes or how many false starts.
   b. Second, learn to do the problem without errors, even if it takes a while.
   c. Third, learn to do the problem correctly and efficiently, without taking too much time or choosing inefficient methods.
10) Remember that understanding while watching someone else is teaching your brain to be a good spectator. But you need to be a good mathlete yourself, not a spectator.

Math 70 Exam #1 Topics List

GC skills on all exercises GC1 through GC 21 ⇒ checked items on list next page

Chapter 1: Sections 3, 6, 7, 8
Chapter 2: Sections 1-5 and 7
Chapter 3: Section 1 only } → GC 22, 23

Word Problems:

Direct translation

Percent
- Basic Percent
- Percent Increase
- Percent Decrease

Consecutive numbers
- integers
- odd integers
- even integers

Geometry
- volume
- area
- perimeter
- angles of triangle
- complementary angles
- supplementary angles
- Pythagorean Theorem

Break-even

Simple Interest

Compound Interest

D=RT uniform motion

Given height function of projectile

Chapter 2: Linear Equations
- y int
- slope } find, interpret
- graph
- write eqn

$y = mx + b$

$y - y_1 = m(x - x_1)$

$m = \dfrac{y_2 - y_1}{x_2 - x_1}$

parallel

perpendicular

VLT

domain

range

piecewise functions

Absolute value equations - graph

function notation, write as function of x

Chapter 3: Solve equation graphically on GC (2 methods)}

Math 70 Graphing Calculator Objectives **rev 5/28/15 M. Carey**
[Throughout this page, GC = Graphing Calculator]

Upon successful completion of Math 70, the student will be able to:

1) Use GC to add, subtract, multiply, divide, calculate absolute value, and find exponents and roots.

2) Use parentheses and the order of operations to complete complicated expressions in one GC entry.

3) Use the GC's fraction command to convert rational numbers in decimal form to fraction form.

4) Understand when a calculated answer is exact or approximate.

5) Store and recall numerical values using memory locations in the GC.

6) Graph a function on the GC using the standard window and, if appropriate, a more meaningful window by adjusting the maximum, minimum, and/or scale values on the x- and y-axes.

7) Produce a graph on paper which neatly and accurately represents the window and scale in the calculator window.

8) Graph two or more functions simultaneously on the GC.

9) Use the GC to generate a table of values for a given function, using x-values which are either individually inputted ("ask") or generated automatically with a specified increment ("auto").

10) Use the GC's root- or zero-finding command to find x-intercepts of a function.

11) Use the GC's evaluate command ("value") to find values of a function , including the y-intercept.

12) Use the graph of a function to verify the domain and range of a function.

13) Use the GC's intersection or zero (root-finding) command to approximate solutions to an equation.

14) Use the GC's matrix commands to find the Reduced Row Echelon Form (RREF) and determine the solution to a system of linear equations from its RREF.

15) Use the GC's fraction command to convert a matrix with rational decimal entries to fraction form.

16) Use the GC to graph exponential and logarithmic functions. {Instructors, please be aware that newer calculators contain change-of-base formula, while older ones do not.)

17) Optional: Use the "trace" command to approximate x- and/or y-coordinates along the graph of a function.

18) Optional: Use the GC's maximum and minimum commands to find relative maximum and minimum values of a function and the x-coordinates where the relative extremes occur.

19) Optional: Use a square window ("zoom square") to view more accurate graphs of perpendicular lines and inverse functions, for example, to graph circles)

20) Optional: Use the GC list commands to calculate entries of a sequence and sum of a finite series.

Math 70 Review Problems Chapters 1-2-3

(1) \$1500 per month plus 5% commission

Pay = 1500 + 0.05 x

x = amount of sales
0.05 = 5% as decimal
0.05 = 5% of sales.

a) In GC $y_1 = 1500 + 0.05x$

[y=]

[TBLSET] start 8000, ATbl = 1000
independent [AUTO]

[TABLE]

sales	\$ 8000	\$9000	\$ 10,000	\$11000	\$12000
pay	\$ 1900	\$1950	\$ 2000	\$ 2050	\$ 2100

b) monthly pay is \$2050
12 months per year = annual pay

$2050 \times 12 = \boxed{\$24{,}600}$

c) Need \$2200 pay per month
Set Pay = 2200 in equation

$2200 = 1500 + 0.05x$ isolate x

$700 = 0.05x$

$\frac{700}{0.05} = x$

$\boxed{x = \$14{,}000 \text{ sales}}$ per month

(2) 55% increase ⇒ percent increase
$a = b + p \cdot b$

b = oldest amount in time = 2008 (unknown)
a = newest amount in time = 2018 = 357000
p = percent, written as decimal = 0.55

$357000 = b + 0.55b$ combine like terms

$357000 = 1.55b$ $1b + 0.55b$

$\frac{357000}{1.55} = b$

$230322.6 = b$

$\boxed{230323 \text{ analysts}}$

Analysts are people ⇒ round to nearest whole analyst.

Math 70

(3) amount of money after compounding quarterly
$\Rightarrow$ Compound interest
$$A = P\left(1+\frac{r}{n}\right)^{nt}$$

A = unknown
P = principal = \$2500
r = interest rate, % written as decimal = 0.035
t = time in years = 10
n = # times compounded per year = 4
quarterly

$$A = 2500\left(1+\frac{.035}{4}\right)^{(4 \times 10)}$$

use ( ) in GC else GC does exponent 4 first and later multiplies result by 10.

$= 3542.272095$

money measured in \$ $\Rightarrow$ round to nearest hundredth
$\approx \boxed{\$3542.27}$

(4) increased linearly $\Rightarrow$ equation of a line $y = mx+b$

21 in 2008 $\quad x = 0$
59 in 2011 $\quad x = 3$

No variables are specified, so you are allowed to make any valid choice, of which there are many
- you can choose any letters you like
- you can make either quantity the independent variable — though the question asks about 2010, which suggests that the year (or time) is a good choice for the independent variable

I choose y = # days lost
x = year, with $x=0$ at 2008 (gives y-intercept!) $b = 21$

find slope $m = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{59-21}{} = \dfrac{38}{3}$

equation of line => $y = \dfrac{38}{3}x + 21$

year 2010 $\Rightarrow x = 2$

$y(2) = \dfrac{38}{3}(2) + 21$

$= \boxed{46\tfrac{1}{3} \text{ days}}$ or $\boxed{46.\overline{3} \text{ days}}$ or even $\boxed{\tfrac{139}{3} \text{ days}}$

Math 70

(5) Unknowns = # tourists expected in U.S. = U
" " " " China = C
" " " " France = F

directly translate each sentence to an equation

$\underset{\text{U.S}}{U} \underset{\text{is predicted}}{=} \underset{\text{million}}{9} \underset{\text{more than}}{+} \underset{\text{France}}{F}$

$\underset{\text{China}}{C} \underset{\text{is predicted.}}{=} \underset{\text{million}}{44} \underset{\text{more than}}{+} \underset{\text{France}}{F}$

Total = add up all 3 countries

$U + C + F = \underset{\text{million}}{332}$

3 equations, 3 unknown variables ☺

substitute 1st eqn for U

substitute 2nd eqn for C

don't forget France!

$(9+F) + (44+F) + F = 332$

$3F + 53 = 332$ combine

$3F = 279$

$F = 93$ $\boxed{\text{France } 93 \text{ million}}$

Substitute Back:

$U = 9 + 93$ $\boxed{\text{U.S. } 102 \text{ million}}$

$C = 44 + 93$ $\boxed{\text{China } 137 \text{ million}}$

(6) Evaluate ⟹ replace x by its value $x = -3.1$

$0.2(-3.1)^3 + 5(-3.1)^2 - 6.2(-3.1) + 3$

can also be done using memory

-3.1 [STO>] [ALPHA] [STO>] X

$0.2X^3 + 5X^2 - 6.2X + 3$

$\underbrace{\text{[ALPHA] [STO>]}}_{\text{means } X}$

$\boxed{64.3118}$

Do NOT ROUND! (not in instructions)

Math 70

7) Simplify = combine like terms, but must be exact!

a) $\underbrace{0.2a + \frac{1}{6}a} - \frac{3}{8}b - 0.75b$

$\left(0.2 + \frac{1}{6}\right)$

$= .3\overline{6}$

w/ bar! exact

OR $= \frac{11}{30}$

using [MATH] 1. [▸frac]

$\left(-\frac{3}{8} - 0.75\right)$

$= -1.125$

terminating = exact

OR $= -\frac{9}{8}$

$= \boxed{.3\overline{6}a - 1.125b}$ or $\boxed{\frac{11}{30}a - \frac{9}{8}b}$

Although it's legal to have a decimal for one and a fraction for the other, it's not very pretty.

b) $(8.3x - 2.9y) - (9.6x - \frac{15}{7}y)$

$= 8.3x - 9.6x - 2.9y + \frac{15}{7}y$ distribute negative

$= \boxed{-1.3x - .75\overline{714285}}$

$= \boxed{\frac{-13}{10}x - \frac{53}{70}y}$

must know the repeating decimal for $\frac{1}{7}$ to write the bar correctly.

when in doubt, use [MATH] [▸frac]

8) Solve for x

a) $Q = xt^2 + vt$

one x appearance ☺ Isolate.

$Q - vt = xt^2$

$\boxed{\frac{Q - vt}{t^2} = x}$

b) $Q = xt^2 + xvt$

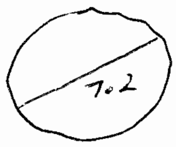
two x appearances → factor out x so it looks like one x.

Math 70

$Q = x(t^2 + vt)$ divide by quantity (t^2+vt)

$$\boxed{\frac{Q}{t^2+vt} = x}$$

(9)



diameter = 7.2 means
radius = $\frac{1}{2}$ diameter = 3.6.

Area = $\pi r^2 = \pi(3.6)^2 = \boxed{12.96\pi \text{ feet}^2 \text{ exact}}$

variables deg2 → exp 2
2-dimensional
ft^2 units

≈ 40.71504079
↑ hundred-thousandth

$\boxed{\approx 40.71504 \text{ feet}^2 \text{ approx}}$

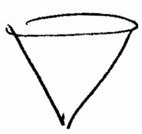
Circumference $= 2\pi r = 2\pi(3.6) = \boxed{7.2\pi \text{ feet exact}}$

deg1 → exp 1
1-dimensional
ft^1 units

≈ 22.61946711

$\boxed{22.61947 \text{ ft approximate}}$

(10)



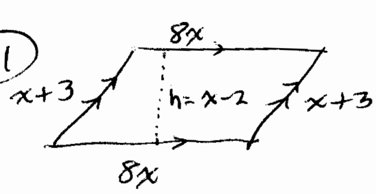
$V = \frac{1}{3}\pi \underbrace{r^2 h}_{\text{deg}3}$ $r = 6$, $h = 2.75$

$V = \frac{1}{3}\pi(6)^2(2.75) = \boxed{33\pi \text{ in}^3 \text{ exact}}$

≈ 103.6725576

$\approx \boxed{103.67 \text{ in}^3 \text{ approximate}}$

(11)



Parallel sides are equal lengths

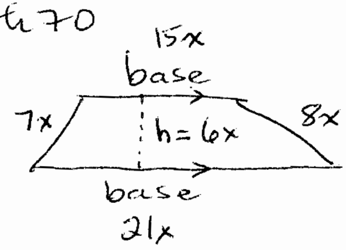
Perimeter = sum of side lengths $= 2(8x) + 2(x+3)$
↳ length, 1D, cm
$= 16x + 2x + 6$
$= \boxed{18x + 6 \text{ cm}}$

Area $= \underbrace{B \cdot H}_{\text{must be perpendicular}} = 8x(x-2) = \boxed{8x^2 - 16x \text{ cm}^2}$

must be perpendicular; degree 2, cm^2

Math 70

(12)



Trapezoid has one pair of parallel sides, called bases.
The height is the (perpendicular) distance between the bases.

perimeter = sum of side lengths

$= 15x + 8x + 21x + 7x$

$= \boxed{51x \text{ meters}}$

$$\text{area} = \tfrac{1}{2}(b_1 + b_2)h$$

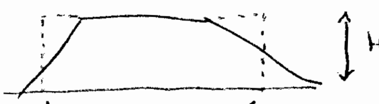
$$= \tfrac{1}{2}(21x + 15x) \cdot 6x$$

$$= 3x\,(\underbrace{21x + 15x}_{\text{like terms}})$$

$$= 3x\,(36x)$$

$$= \boxed{108x^2 \text{ meters}^2}$$

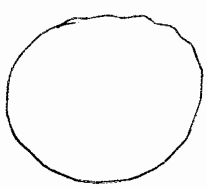
$\tfrac{1}{2}(b_1+b_2)$ is the average of the two bases



$\tfrac{1}{2}(b_1+b_2) = B$

which can be seen as a rectangle.
Area = B H.

(13)



C = 78.5 ft ← ft, 1-dim

each dog needs 60 ft² ← ft², 2-dim.

Need area $A = \pi r^2$

Need radius r.

$$C = 2\pi r$$

$$78.5 = 2\pi r$$

$$\frac{78.5}{2\pi} = r$$

set known circumference = formula
solve for r

Do not round (yet!) or we'll have round-off error.

$$A = \pi r^2$$

$$A = \pi\left(\frac{78.5}{2\pi}\right)^2 = \pi\left(\frac{39.25}{\pi}\right)^2$$

simplify $\frac{78.5}{2} = 39.25 = \frac{157}{4}$

$$= \frac{\pi\,(39.25)^2}{\pi^2}$$

cancel (divide out) one π

$$= \frac{(39.25)^2}{\pi}$$

$$= \frac{1540.5625}{\pi} = \text{area, exactly.}$$

Math 70.

Each dog $60 \text{ ft}^2/\text{dog}$ | Total $\frac{1540.5625}{\pi} \text{ft}^2$ | # dogs ?

$$\# \text{dogs} = \frac{\text{Total area ft}^2}{\left(\text{area per dog } \text{ft}^2/\text{dog}\right)} = \text{ft}^2 \div \frac{\text{ft}^2}{\text{dog}} = \text{ft}^2 \cdot \frac{\text{dog}}{\text{ft}^2} = \text{dog}$$

$$= \frac{\left(\frac{1540.5625}{\pi} \text{ft}^2\right)}{\left(60 \text{ ft}^2/\text{dog}\right)}$$

$$= \frac{1540.5625}{\pi} \div \frac{60}{1}$$

$$= \frac{1540.5625}{\pi} \cdot \frac{1}{60}$$

$$= \frac{1540.5625}{(60\pi)} \quad \text{exact}$$

$$\approx 8.172937901 \quad \text{approximate}$$

no parts of dogs ☹

$$\approx \boxed{8 \text{ dogs}}$$